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# DIRECT NUMERICAL SIMULATION OF THE FULLY DEVELOPED OPEN-CHANNEL FLOW AT SUBCRITICAL FROUDE NUMBERS

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**Abstract.** Direct numerical simulation(DNS) of a fully-developed open-channel flow has been carried out for subcritical Froude numbers at a fixed friction-velocity Reynolds number of 180. The free surface is approximated by small-amplitude wave theory. The results are presented emphasizing the effects of the Froude number on turbulence quantities related to the free surface fluctuations. The amplitude of the free-surface fluctuation increases as the square of the Froude number. While statistical quantities involving the vertical velocity component are most influenced by the Froude number but only in the region close to the free surface, quantities related to the pressure fluctuations are influenced over much wider region.

## 1. Introduction

Turbulent flows in open-channels are important in engineering and environmental applications, but their understanding and prediction are not as satisfactory as those for flows in closed ducts. Turbulence mechanisms near a free surface have been investigated both experimentally and numerically(e.g. Kumar *et al.*, 1998 and Nagaosa, 1999), but no specific methods of modeling in practical engineering calculations are established. It may be noted that the recent advancements of the wall turbulence modeling have been made possible by detailed databases obtained by DNS. DNS for open-channel flows have been reported, but most of these simulations assume that the free surface is a rigid slip surface and its vertical movement is neglected(e.g. Nagaosa, 1999). In the DNS conducted by Komori *et al.*(1993), the Froude number is so small that there are no recognizable differences from the slip-surface flow. Borue *et al.*(1995) applied the linearized free-surface boundary conditions and obtained the dynamics associated with

the turbulence/interface interactions. They did not provide the type of database that would be required to refine and validate turbulence models for open-channel flows. In the present paper, we report the results of DNS of a fully developed turbulent flow in a two-dimensional open channel with deforming free surface approximated by the small-amplitude wave assumption using a finite difference method. We obtain a detailed database of the mean flow and the turbulence quantities for subcritical Froude numbers.

## 2. Basic Equations and Boundary Conditions

We consider a fully developed incompressible open-channel flow over a flat smooth floor as shown in Fig.1. The governing equations are the Navier-Stokes equations and continuity equation for incompressible fluid with gravitational acceleration  $g$ . The boundary conditions are the no-slip condition on the bottom floor ( $x_2 = 0$ ), and the free-surface conditions as described in the following. The flow field is supposed to extend infinitely in the streamwise ( $x_1$ ) and the cross-flow ( $x_3$ ) directions and the periodic boundary conditions are applied at the boundaries of the finite computational domain. We denote the time mean of instantaneous quantity  $\hat{f}$  by  $\bar{f}$  or  $F$ , the deviation from the mean by  $f$ , the root mean square by  $f^{rms}$  and the value at  $x_2 = a$  by  $\hat{f}|_a$ . The free surface movement is assumed mild and it can be described by a smooth continuous surface which deforms continuously so that its position can be represented by a single-valued continuous function by  $x_2 = \hat{h}(x_1, x_3, t)$ . Function  $\hat{h}$  satisfies the kinematic condition

$$\frac{\partial \hat{h}}{\partial t} + \hat{u}_1|_{\hat{h}} \frac{\partial \hat{h}}{\partial x_1} + \hat{u}_3|_{\hat{h}} \frac{\partial \hat{h}}{\partial x_3} = \hat{u}_2|_{\hat{h}}, \quad (1)$$

where  $\hat{u}_i$  is the instantaneous velocity component in  $x_i$  direction. If we assume that the fluid above the free surface has negligible density and the pressure is constant zero, and that the surface tension can also be neglected, the dynamic conditions are that both normal and tangential stresses on the instantaneous free surface are zero. If the displacement of the instantaneous free surface from the mean position and the slopes of the free surface is small, the quantities on it may be replaced by the values on the mean position and the normal and tangential directions may be replaced by the vertical and horizontal directions, respectively, then the first approximation gives

$$\frac{\partial \hat{h}}{\partial t} + \frac{\partial}{\partial x_1} (\hat{u}_1|_H \hat{h}) + \frac{\partial}{\partial x_3} (\hat{u}_3|_H \hat{h}) = \hat{u}_2|_H, \quad (2)$$

$$\nu \left( \frac{\partial \hat{u}_1}{\partial x_2} + \frac{\partial \hat{u}_2}{\partial x_1} \right) \Big|_H = 0, \quad \nu \left( \frac{\partial \hat{u}_3}{\partial x_2} + \frac{\partial \hat{u}_2}{\partial x_3} \right) \Big|_H = 0, \quad (3)$$

$$\hat{P}|_H = -g_2 h + 2\nu \frac{\partial \hat{u}_2}{\partial x_2} \Big|_H, \quad (4)$$

where  $\hat{P} = \hat{p} - g_2 h$ ,  $\hat{p}$  is the instantaneous pressure divided by the density,  $g_2$  is the component of gravitational acceleration in  $x_2$  direction and  $\nu$  is the kinematic viscosity. Since these boundary conditions are all in terms of the quantities at the mean position of the free surface, the flow calculations can be made with the fixed grid in the region  $x_2 < H$ .

### 3. Numerical Methods

The governing equations are solved by a finite difference technique based on the SMAC method on a Cartesian staggered grid. The spatial derivatives are discretized using the second-order conservative difference scheme and time advancing is done by the second-order Adams-Bashforth method. The position of the free surface  $h$  is solved by discretizing the spatial derivative terms in Eq.(2) using the fifth-order upwind-shifted interpolation(USI) scheme(Kajishima, 1994) and time advancing by the third-order Adams-Bashforth method. The Poisson equation for pressure is solved by Fourier-transforming the equations in the horizontal directions and the resulting equation is solved by the tri-diagonal algorithm. The calculations were conducted for the Reynolds number  $Re_\tau$  based on the friction velocity  $u_\tau$  and the mean flow depth  $H$  of 180, which is the same as the closed-channel flow simulation of Moser *et al.*(1999)(MKM) and Yokojima & Nakayama(2000)(YN) for a similar flow with free-slip boundary condition (hereafter referred to as the 'slip channel'). The Froude numbers based on the average velocity and the flow depth are 0.3, 0.6 and 0.9. The number of grid points is  $128^3$ ; the spatial resolutions are  $\Delta x_1^+ = 9$ ,  $\Delta x_2^+ = 0.27 - 2.71$ ,  $\Delta x_3^+ = 4.5$ . Here the superscript  $+$  refers to the nondimensionalized quantities by  $u_\tau$  and  $\nu$ . With a time increment of  $\Delta t^+ = 0.0114$ , about 200,000 time steps are used to obtain the well-converged turbulence quantities averaged over space and time.

### 4. Results and Discussion

Fig.2 shows properties of the calculated instantaneous flow field for the slip-channel flow at  $Fr = 0$  and the present fluctuating free surface computation for  $Fr = 0.6$ . The figures on the top are the samples of instantaneous flow represented by the iso-surfaces of the streamwise velocity  $u_1$ . The instantaneous position of the free surface  $\hat{h}$  and the velocity vectors projected in the mean free-surface position are shown in the middle and at the bottom, respectively. The well-known streak structure of the wall turbulence is seen in both cases near the bottom floor and the overall simulation method ap-

pears satisfactory. In  $Fr = 0$  case,  $\hat{h}$  is not available and the instantaneous pressure at the free surface  $\hat{p}|_H$  is shown instead. The surface pressure and  $h$  appear to show very a similar trend. The free surface fluctuation and the velocity vectors do not seem to have any preferred directionality and their scales appear to be much larger than the viscous scales of the near-wall flow. These tendencies agree with the results of Thomas & Williams(1995)(TW) who reported preliminary results of a direct simulation using 'volume of fluid' method to track the movement of the free surface. The velocity vectors on the free surface indicate that the flow is generally divergent in the horizontal plane where the upwelling bulges with positive  $h$  are seen and convergent in the area the free surface is lower than the average position.

The profiles of computed RMS surface fluctuation are compared with experimental results of Nakayama *et al.*(2000)(EXPN00) and Nakayama (1997)(EXPN97) in Fig.3. The Reynolds numbers of the experiments are indicated in terms of the bulk Reynolds number defined by the average velocity and the mean depth  $H$ . It shows that the present DNS results are a little lower than the measurements but show the same increasing trends with respect to  $Fr$  and are very close to the results of TW. Though not shown, the computed skewness and flatness factors of  $h$  also agree well with those of measurements, which indicates that the small-amplitude approximation used here is appropriate for the DNS of subcritical flows.

Fig.4 shows the computed mean velocity profiles compared with the closed-channel and slip-channel flow simulations. It shows that the velocity profile of the open-channel flow follows the logarithmic distribution up to a point very close to the free surface and it is not influenced very much by the free-surface movements. The distributions of the RMS fluctuations of velocity components and the Reynolds shear stress are shown in Fig.5. It is seen that the distributions of all stress components are very close to the closed-channel results of Moser *et al.*(1999) over most of the channel. The open-channel flow results deviate from the closed channel results in the region  $120 < x_2^+ < 180$ . Fig.6 is an enlarged plot of  $u_2^{rms}$  which is the quantity most sensitively affected by the free surface and  $Fr$ .  $u_2^{rms}|_H$  increases with roughly square of  $Fr$ , which is the same as  $h^{rms}$ . Fig.7 is a plot of the RMS pressure fluctuation. Unlike the velocity fluctuations,  $p^{rms}$  is seen to be influenced by the free surface and the value of  $Fr$  throughout the entire channel depth. Since the Reynolds stress component  $\overline{u_2^2}$  is most influenced by the Froude number, the terms in the transport equation for this stress are shown in Fig.8. Fig.8(a) is the distribution across the entire channel and Figs.8(b)-(d) show enlarged plots near the free surface for  $Fr = 0.3, 0.6$  and  $0.9$ , respectively, each compared with  $Fr = 0$  case. It is seen that near the free surface the pressure-strain term, which works like a generation for  $\overline{u_2^2}$ , drops and the pressure diffusion increases. It is further

seen in the enlarged plots of Figs.8(b)-(d) that the pressure diffusion term changes sharply from loss to gain within about 20 viscous units from the free surface. Its distribution and that of the turbulent diffusion appear to be influenced by  $Fr$  over at least upper half of the channel.

### 5. Conclusions

Direct numerical simulation of a fully developed turbulent flow in a two-dimensional open channel has been successfully performed. The present results with the assumption of small displacement of the free surface from its mean position appear to represent the open-channel flow with subcritical Froude numbers properly, and show the effects due to the moving free surface. Most of the quantities including the mean velocity profiles and the Reynolds stresses in the horizontal directions are not influenced by the free-surface movements. The vertical velocity fluctuation and the pressure fluctuation are the quantities influenced by the free surface. The vertical fluctuation near the free surface increases with  $Fr$  in the region very close to the free surface, while the pressure fluctuation is influenced over almost entire depth of the channel. This influence appears to be effected by the readjustment by the pressure diffusion.

### References

- Borue, V., Orszag, S.A. and Staroselsky, I. (1995) Interaction of surface waves with turbulence: direct numerical simulations of turbulent open-channel flow, *J. Fluid Mech.*, Vol.286, pp.1-23.
- Kajishima, T. (1994) Upstream-shifted interpolation method for numerical simulation of incompressible flows, *Trans. JSME B*, Vol.60 No.578, pp.3319-3326 (in Japanese).
- Komori, S., Nagaosa, R., Murakami, Y., Chiba, S., Ishii, K. and Kuwahara, K. (1993) Direct numerical simulation of three-dimensional open-channel flow with zero-shear gas-liquid interface, *Phys. Fluids A*, Vol.5 No.1, pp.115-125.
- Kumar, S., Gupta, R. and Banerjee, S. (1998) An experimental investigation of the characteristics of free-surface turbulence in channel flow, *Phys. Fluids*, Vol.10 No.2, pp.437-456.
- Moser, R.D., Kim, J. and Mansour, N.N. (1999) Direct numerical simulation of turbulent channel flow up to  $Re_\tau = 590$ , *Phys. Fluids*, Vol.11 No.4, pp.943-945.
- Nagaosa, R. (1999) Direct numerical simulation of vortex structures and turbulent scalar transfer across a free surface in a fully developed turbulence, *Phys. Fluids*, Vol.11 No.6, pp.1581-1595.
- Nakayama A., Nakase, Y., Yokojima, S. and Fujita, I. (2000) Improvements of two-equation turbulence model with surface fluctuation used as a parameter for calculation of open-channel flows, *J. Applied Mech. JSCE*, Vol.3, pp.745-752 (in Japanese).
- Nakayama, T. (1997) Turbulent structures and characteristics of coherent vortices near the free-surface, Master's thesis, Kyoto University (in Japanese).
- Thomas, T.G. and Williams, J.J.R. (1995) Turbulent simulation of open channel flow at low Reynolds number, *Int. J. Heat Mass Transfer*, Vol.38 No.2, pp.259-266.
- Yokojima, S. and Nakayama, A. (2000) Evaluation of turbulence statistics and their budgets in an open-channel flow using direct numerical simulation, *J. Applied Mech. JSCE*, Vol.3, pp.753-762 (in Japanese).

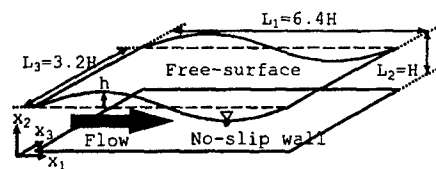


Figure 1. Flow configuration of open-channel flow.

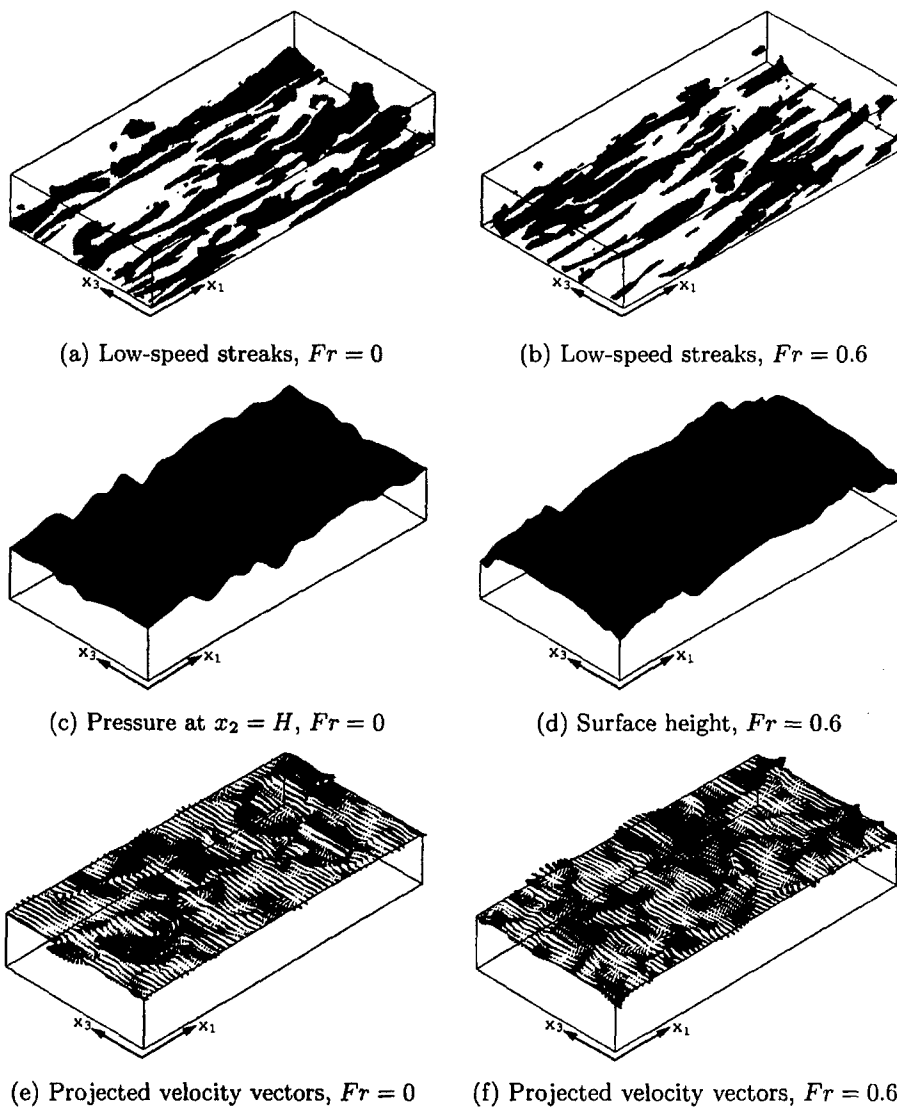


Figure 2. Instantaneous flow features.

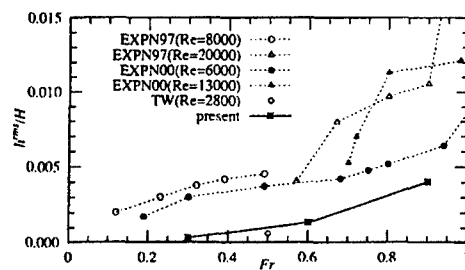


Figure 3. RMS fluctuations of  $h$ .

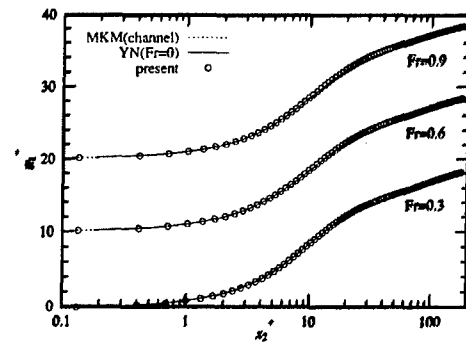
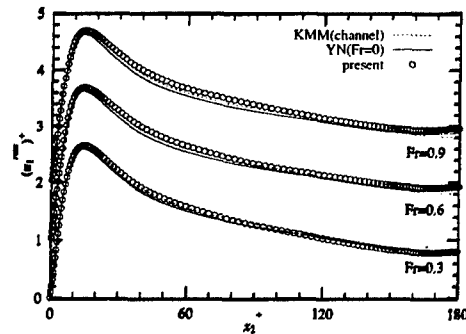
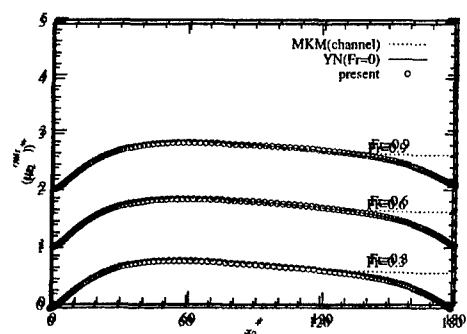


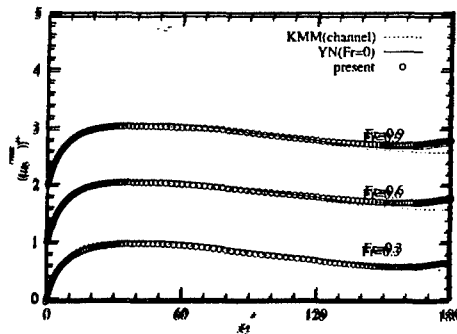
Figure 4. Mean velocity profiles.



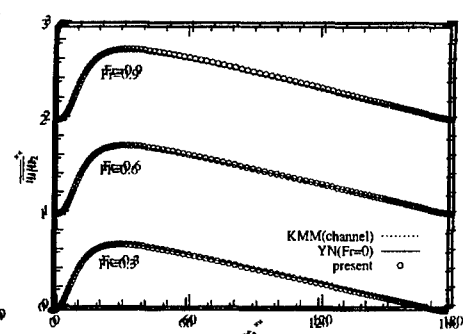
(a)  $u_1^{\text{rms}}$



(b)  $u_2^{\text{rms}}$



(c)  $u_3^{\text{rms}}$



(d)  $\overline{u_1 u_2}$

Figure 5. Profiles of turbulence intensities and Reynolds shear stress.



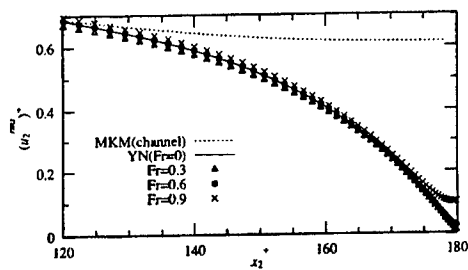
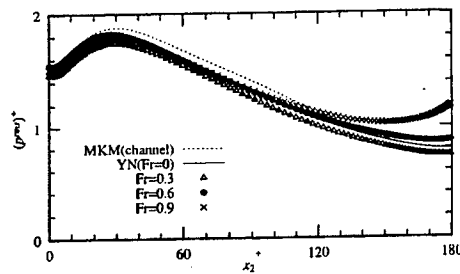
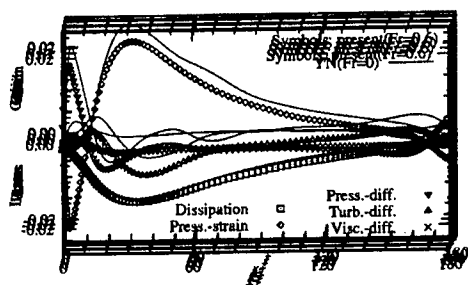
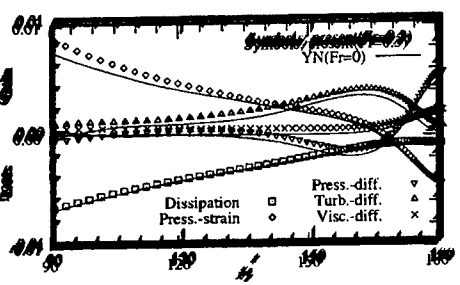
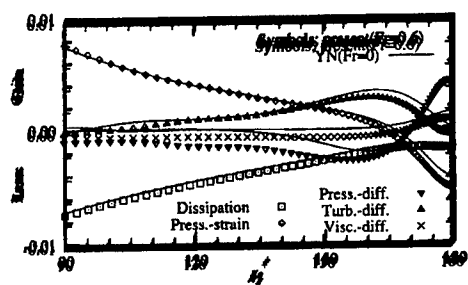
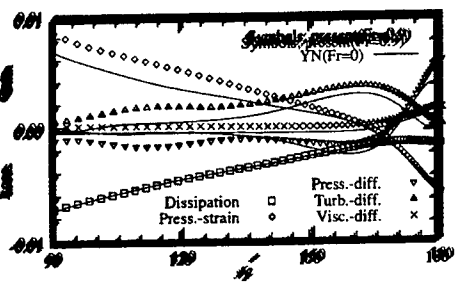
Figure 6.  $u_2^{\text{rms}}$  near free surface.

Figure 7. RMS pressure fluctuation.

(a) Entire depth,  $Fr = 0, 0.6$ (b) Near free surface,  $Fr = 0, 0.3$ (c) Near free surface,  $Fr = 0, 0.6$ (d) Near free surface,  $Fr = 0, 0.9$ Figure 8.  $\overline{u_2^2}$  budget.